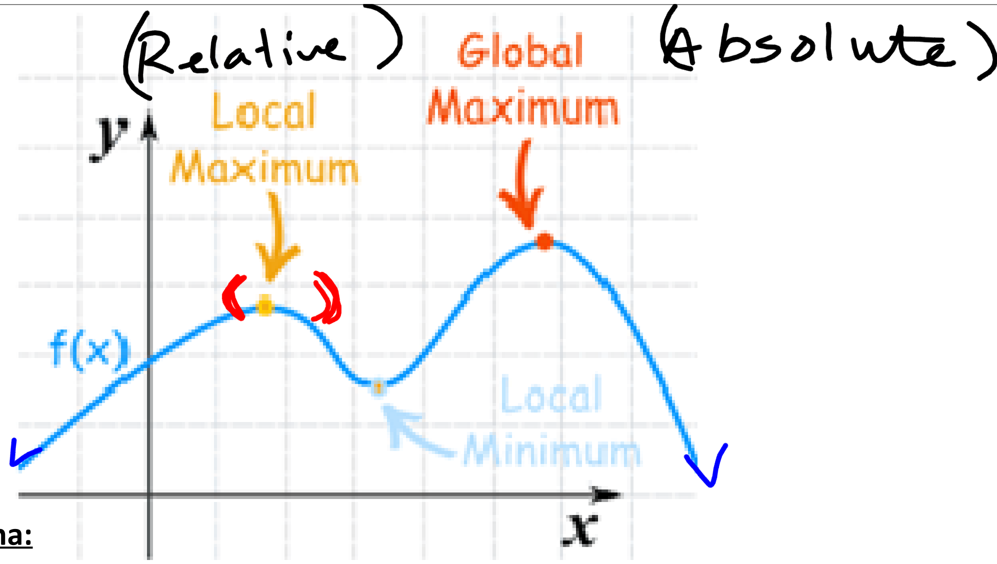
4-1 Extreme Value of a Function

Learning Targets

I can identify relative and absolute extrema from a graph.

I can apply the Extreme Value Theorem to identify absolute extrema on a closed interval.

I can identify an critical points in a function.



Absolute Extrema:

Let f(x) be a function with domain D. Then

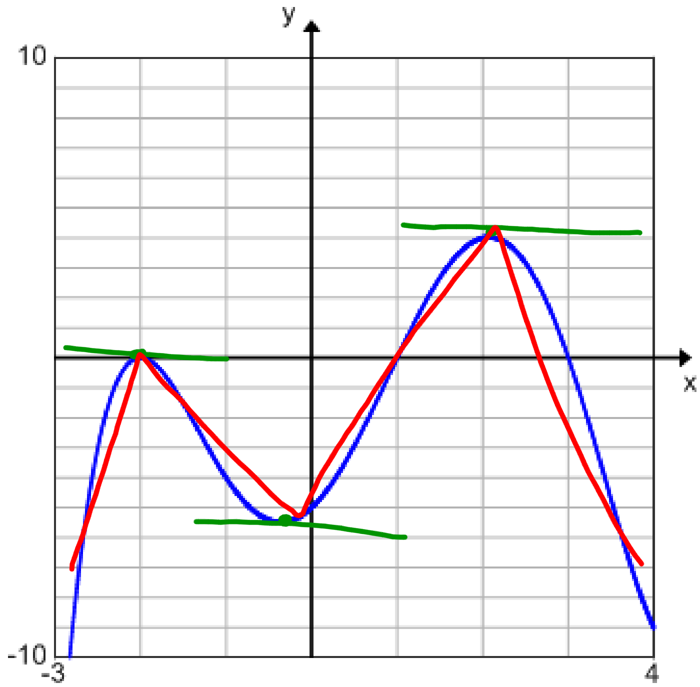
- f(c) is the absolute maximum value on D if and only if $f(x) \le f(c)$ for all x in D.
- f(c) is the absolute minimum value on D if and only if $f(x) \ge f(c)$ for all x in D.

Local Extrema:

Let f(x) be a function defined on an open interval containing c. Then

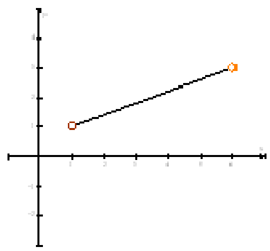
- f(c) is a local maximum value if and only if $f(x) \le f(c)$ for all x in an open interval containing c.
- f(c) is a local minimum value if and only if $f(x) \ge f(c)$ for all x in an open interval containing c.

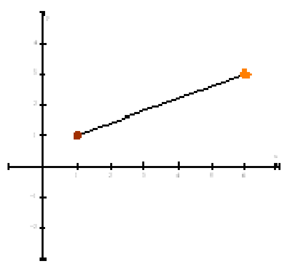
Derivatives help us locate max's and min's because the slopes of the tangent line at a max or min value = 0.

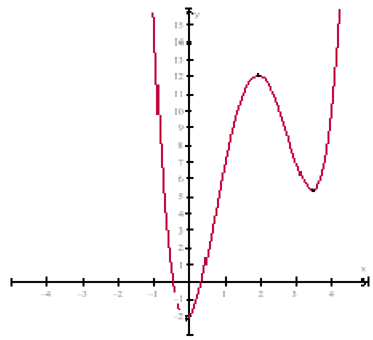


1. Der = 0
2. Der is undef
3. endpt

On a closed interval, the max's or min's can occur at interior points OR at an endpoint.





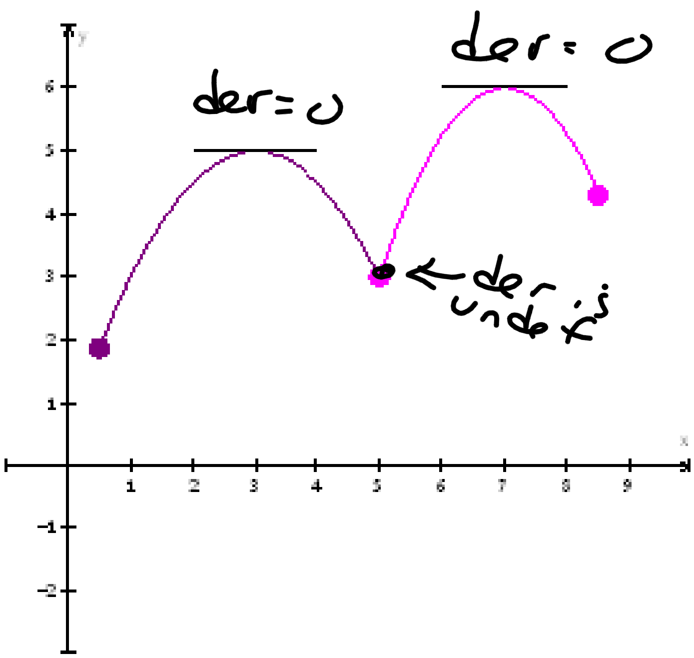


A max or min can exist at an endpoint on a function defined on a closed interval even though f'(x) does not exist at that point.

The same is not true on an open interval. An abs max or a min may or may not exist. A max or min cannot exist at an endpoint on an open interval because there is not an endpoint.

Critical Point

A point is a critical point if f'(x)=0 (~~max or min~~ horiz tangent line) or if f'(x) doesn't exist (sharp corner, discontinuity, vertical tangent line).



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Decide if the extreme value theorem applies to the given function. Find the absolute extrema (if they exist).

candidates

$x = -2, 2$

1. $f(x) = e^{2x}, -2 \le x \le 2$

$f' = e^{2x} \cdot 2$

$f' = 2e^{2x}$

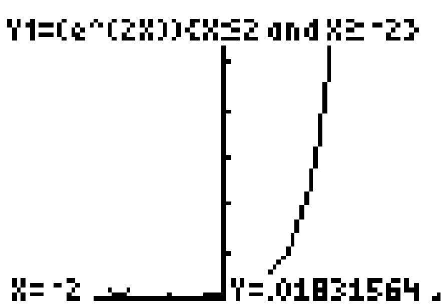
$0 = 2e^{2x}$

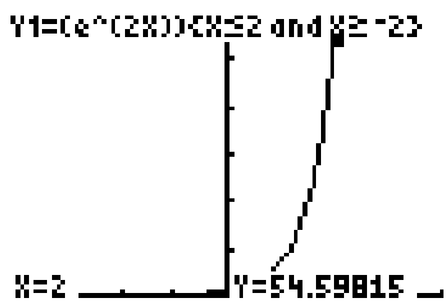
$0 = e^{2x}$

Der $\neq 0$ Der $\neq$ undef

Abs Max = e^{4} @ $x = 2$

Abs Min = $\frac{1}{e^{4}}$ @ $x = -2$





2. $y = 3x^2 - 4x + 12$ on $[-2, 4]$

candidates

endpts: $x = -2, 4$

der=0 $x = 2/3$

der=undef N/A

@ $x = -2$ $y = 32$

@ $x = 4$ $y = 44$

@ $x = 2/3$ $y = 10\,2/3$

$y' = 6x - 4$

$0 = 6x - 4$

$x = 2/3$

Abs Max = 44

Abs Min = $10\,2/3$

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$(-1)^{2/3}$

3. $g(x) - x^{\frac{2}{3}}$ on $[-1,3]$

candidates:

endpts: $x = -1, 3$

der=0: N/A

der=undef: $x = 0$

@ $x = -1$ $y = 1$

@ $x = 0$ $y = 0$

@ $x = 3$ $y = \sqrt[3]{9}$

$g' = \frac{2}{3}x^{-1/3}$

$g' = \frac{2}{3\sqrt[3]{x}}$

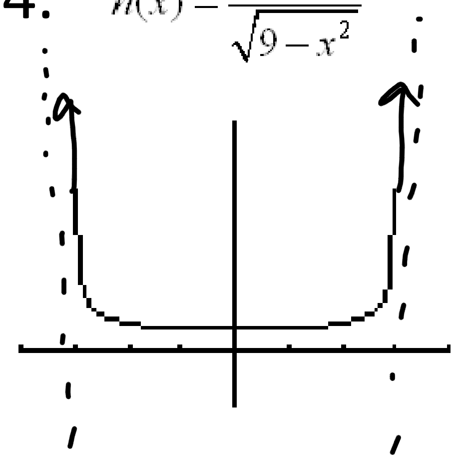
$0 \neq \frac{2}{3\sqrt[3]{x}}$

Abs Max $= \sqrt[3]{9}$

Abs Min $= 0$

4. $h(x) = \frac{2}{\sqrt{9-x^2}}$

Domain $-3 < x < 3$



NO

$\boxed{\text{Abs Min} = 2/3}$

$K(x) = \sqrt{9-x^2}$ Domain $-3 \le x \le 3$

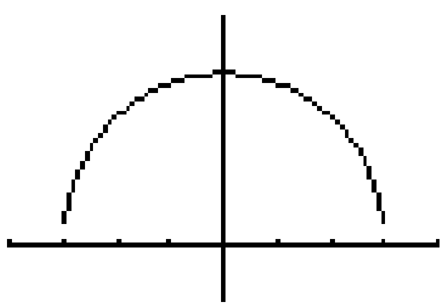
$x = 3, -3, 0$

@ $x = 3$ $K(3) = 0$

@ $x = -3$ $K(-3) = 0$

@ $x = 0$ $K(0) = 3$

$\boxed{\begin{array}{l}\text{Abs Max} = 3 \\ \text{Abs Min} = 0\end{array}}$



Homework

p. 193 #1-11, 13, 14, 17, 18, 25, 31, 32, 45-50

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